$$\frac{I}{jk_0Y_0} = \frac{\gamma^2 - \gamma_0^2}{\gamma_0}\int_S \bar{a}_z \cdot \bar{e}_t \times \bar{h}_t dS - Z_m \oint_C |\bar{\tau} \cdot \bar{J}|^2 - |\bar{a}_z \cdot \bar{J}|^2 dl. \quad (31)$$

Expansion (9) can now be substituted into (31) and taking note of the fact that if h_{tn} is real then h_{zn} is purely imaginary, (31) becomes proportional to

$$\sum_{k=1}^{N}\sum_{r=1}^{N} a_k a_r \left[\frac{\gamma_0^2 - \gamma^2}{\gamma_0(1+j)} \mathcal{P}_{kr} + P_{kr}\right]. \quad (32)$$

This quantity is of the same form as (16) except here $\mathcal{P}_{kr}$ and P_{kr} are Hermitian. The matrix eigenvalue problem which occurs when the variation of (32) with respect to the a_i is set to zero must also have real eigenvalues. One then finds

$$\frac{\gamma_0^2 - \gamma^2}{\gamma_0(1+j)} \approx -2\,\frac{(\Delta\beta - j\alpha)}{(1-j)}$$

must be real, so that $\Delta\beta = \alpha$ and the resulting matrix eigenvalue problem is just (11).

Conclusions

By means of the variational approach, the following general properties of degenerate modes in lossy waveguides and cavities have been shown: 1) The degenerate modes of the ideal structure split into an equal number of nondegenerate modes in the lossy structure. 2) The split occurs such that each of these new modes possess the orthogonality properties of a nondegenerate mode. 3) Each of these new modes individually satisfies the single mode power loss approximation. 4) In the case of the cavity the shift in resonant frequency due to losses is equal to the damping factor and for the waveguide the shift in the propagation factor is equal to the attenuation constant.

In any particular example, the actual calculation of the damping factor or Q of a cavity or the attenuation constant of a waveguide has been systematized into the solving of the determinant of a matrix eigenvalue problem, if desired the field distribution in the lossy structure can also be found by solving for the eigenvectors of the problem, thus obtaining the expansion coefficients of the ideal modes.

Correspondence

Comment on "A Simple Method for Measuring the Phase Shift and Attenuation through Active Microwave Networks"*

In his recent letter[1] on the measurement of phase shift and attenuation of active two ports Alday has submitted material which the author believes should not remain unchallenged. His notion of the addition of power in his system is an erroneous one, so that what he claims to be a measure of attenuation is in fact a measure of a quantity which is not an attenuation at all. It is, as will be seen, a quantity which qualitatively follows the "ups and downs" of a peculiar attenuation.

When the left- and right-hand ports of the network to be measured in Alday's Fig. 1 are designated as 1 and 2, and the network is described by its scattering matrix (S), the wave which reaches the detector of the "attenuator loop" has an amplitude which is proportional to $|S_{11}|+|S_{21}|$ after the indicated phase shifter adjustment. Now, assuming square law detection, one has $|S_{11}|^2+|S_{21}|^2+2|S_{11}S_{21}|$. When the network is lossless (as distinguished by primes) the sum $|S_{11}'|^2+|S_{21}'|^2$ remains constant at unity; however, $|S_{11}'S_{21}'|$ depends on the details of the network so that Alday's measured "attenuation" is

$$10\log_{10}\left[\frac{1+2|S_{11}'S_{21}'|}{|S_{11}|^2+|S_{21}|^2+2|S_{11}S_{21}|}\right]\text{db.} \quad (1)$$

Assuming $|S_{11}'S_{21}'|$ constant, this quantity, in a vague sense, behaves like

$$10\log_{10}\left[\frac{1}{|S_{11}|^2+|S_{21}|^2}\right]\text{db,} \quad (2)$$

where $1/(|S_{11}|^2+|S_{21}|^2)$ is the ratio of available power (to a matched load) to power *not* dissipated by the network (the sum of the "reflected" and "transmitted" powers). One can only infer that Alday's intention was to measure the peculiar attenuation given by (2) or the related quantity

$$10\log_{10}\left[\frac{1}{1-(|S_{11}|^2+|S_{21}|^2)}\right]\text{db,} \quad (3)$$

which would give, in decibels, the portion of available power dissipated by the two port under test. In any case, Mr. Alday owed it to his readers to define his terms.

H. M. Altschuler
Polytechnic Inst. of Brooklyn
Brooklyn, N. Y.

* Received May 22, 1962.

[1] J. R. Adlay, IRE Trans. on Microwave Theory and Techniques (*Correspondence*), vol. MTT-10, p. 143; March, 1962.

Measurement of Impedance at Frequencies above 300 Mc*

I wonder if any of your readers can help me to find out whether the method described below has ever been proposed in the literature to measure the output impedance of signal sources in the frequency range above 300 Mc. Although it is realized that this method is in close relationship with Chipman's method,[1] to the author's best knowledge it has not been proposed for the measurement of active two-terminal impedances, *i.e.*, signal generator output impedances in particular.

The equipment needed is a sliding short-circuit stub with a pickup loop mounted at the short circuit. A quantity proportional to the current in the short circuit is read on a detector which can be any distance (in electrical length) from the pickup loop. If the subscripts of the absolute values of currents denote the corresponding electrical distance of the position of the effective short circuit from the plane where the generator impedance is to be measured, then it can be shown that the generator impedance $Z_g = Z'$

* Received August 29, 1962.

[1] R. A. Chipman, "A resonance-curve method for absolute measurement of impedance at frequencies of the order of 300 Mc/s," *J. Appl. Phys.*, vol. 10; January, 1939.